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An economic decision model for engineers.

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AN ECONOMIC DECISION MODEL
FOR ENGINEERS

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An Economic Decision Model For Engineers

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by

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PREFACE

This paper was written at the specific request of the Department of Nuclear Engineering for the purpose of introducing its students to some of the concepts of engineering economy. The concepts presented are rather elementary to the business student; however, to the engineer with no background in business, they are quite new. It is hoped that through this introduction the engineering student will gain some insight into engineering economics and its importance to him as an engineer, and thus encourage him to pursue this course of study in more depth.

All equations presented with the exception of the final two equations, (6-10) and (6-11), were obtained from reference material as noted. The final two equations, which represent the decision model, are original equations derived by the author.

ACKNOWLEDGMENT

The author wishes to express his sincere appreciation to Dr. Jack C. Hayya and Dr. Warren F. Witzig for their objective appraisal and technical review of the manuscript. Recognition should also be given to the United States Navy for providing the opportunity and financial support for the author to conduct his studies.

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I. INTRODUCTION

The purpose of this paper is to present an analytical method that will aid the engineer or manager in the selection of alternative projects for investment. The judgment of risk versus potential return on investment cannot be made solely on the basis of an objective economic analysis. However, such an analysis will provide an important and possibly decisive input to the selection process.

Actual decisions are generally made within the context of a limited and highly approximate abstraction of the actual situation. In order to accurately select between alternative projects, a method of value measurement by which all alternatives can be compared would be useful.

The business world of today is profit motivated; therefore, an engineer must be concerned with not only the technical considerations of a project or design, but also with its economic feasibility. Both the technological and economic considerations must be understood in applying science to useful ends. In addition, cost-benefit analysis which considers and balances the environmental effect of the alternatives, to the extent that these considerations can be quantified, may also be examined. Thus, the engineer's technical capabilities should be supplemented with the ability to make an economic appraisal of his design or recommendations.

One method of analyzing the economic feasibility of technical alternatives, leading to a decision or recommendation for a particular alternative, is presented in this paper.

II. THE TIME VALUE OF MONEY

The rent paid for the use of a building is essentially the same as interest paid for the use of money. To the lender, the interest he receives must cover the cost of making the loan. These costs include the cost of the opportunities foregone, since the money is no longer available for other purposes; and the service cost of handling the loan. To the borrower, the interest charge represents an expense for the use of the money[3].

Money does no good until it is spent. The time value of money means that you should be rewarded if you are willing to lend or invest your money and thereby postpone other opportunities for its use.

Assuming that one is satisfied with a yearly interest rate of 5%, then $1.00 received today is equivalent in desirability to $1.05 being received a year from now. Conversely, $1.00 received a year from now is worth only $.95 today. In other words, the present worth of $1.00 received a year from now is $.95.

The preceding simple example illustrates the idea of the time value of money. In analyzing the economic aspects of an engineering problem, the receipts and dispersments of capital will occur at various times and in varying amounts over the life of the asset. In such a situation, reducing these economic values to a common base by using the concept of interest is essential for the proper selection among alternative courses of action. How the concept of interest is employed will now be discussed with the aid of the following abbreviations:

I = Interest charged or received

i = Interest rate

n = Number of years

P = Present sum, principal, invested cost, or present worth of future money

S = Future sum of money

Simple Interest

An interest rate is a ratio of the rent paid for the use of a sum of money to that amount of money, over a period of time, usually a year. When simple interest is charged, interest is earned only on the money loaned, and is proportional to the duration of the loan:

$$I = Pni$$

If \$10,000 was borrowed at a simple annual interest rate of 10% for five years, then the interest paid and the total amount to be paid at the end of five years would be:

$$\begin{aligned} I &= \$10{,}000(5)(.1) \\ &= \$\ 5{,}000 \\ P + I &= \$15{,}000 \end{aligned}$$

Throughout this paper, interest calculations will be based on the assumption that time is divided into discrete periods, usually one year, and that the interest rate is stated as an annual amount.

Compound Interest

When compound interest is used, the money earned at the end of each interest period either becomes due at that time or earns interest upon itself. In the example presented under simple interest, the total interest of \$5,000 would be paid in five equal payments of \$1,000 each. This money could then be reinvested at the same interest rate for the

remaining term of the loan, or it could be used for some other purpose. Reinvesting would result in the total interest earnings and total payment given in Table 2-1.

Table 2-1

Effect of Compound Interest

Year	Amount Invested Beginning of Year	Interest Earned End of Year	Amount Invested End of Year
1	$10,000	$1,000.00	$11,000.00
2	11,000	1,100.00	12,100.00
3	12,100	1,210.00	13,310.00
4	13,310	1,331.00	14,641.00
5	14,641	1,464.10	16,105.10

Compound Amount Factor

With interest permitted to compound as in Table 2-1, the relationship between the original investment, P, and the future sum, S, can be derived by substituting general terms for the quantities in Table 2-1. This has been done in Table 2-2.

Table 2-2

Derivation of the Compound Amount Factor[3]

Year	Amount Invested Beginning of Year	Interest Earned End of Year	Amount Invested End of Year
1	P	iP	$P(1+i)$
2	$P(1+i)$	$iP(1+i)$	$P(1+i)^2$
3	$P(1+i)^2$	$iP(1+i)^2$	$P(1+i)^3$
4	$P(1+i)^3$	$iP(1+i)^3$	$P(1+i)^4$
5	$P(1+i)^4$	$iP(1+i)^4$	$P(1+i)^5 = P(1+i)^n$

The quantity $(1+i)^n$ is known as the compound amount factor, and is designated CAF(i,n). This factor is used to express the equivalence between a present and future sum at interest rate i, for n years:

$$\mathrm{CAF}(i,n) = (1+i)^n \tag{2-1}$$

$$S = (P)\mathrm{CAF}(i,n) \tag{2-2}$$

$$S = P(1+i)^n \tag{2-3}$$

Using the example of Table 2-1:

$$\begin{aligned} S &= \$10{,}000(1+.1)^5 \\ &= \$10{,}000(1.6105) \\ &= \$16{,}105 \end{aligned}$$

The various factors discussed in this chapter can be obtained from tables found in many business textbooks such as in References 1, 3, 5, and 8.

Present Worth Factor

Solving equation (2-3) for P yields:

$$P = S(1/(1+i)^n) \tag{2-4}$$

Where the quantity $1/(1+i)^n$ is known as the present worth factor; and it is designated PWF(i,n). This factor is used to express the equivalence between a future sum, S, and a present sum, P, at an interest rate i, for n years:

$$PWF(i,n) = 1/(1+i)^n \tag{2-5}$$

$$P = (S)PWF(i,n) \tag{2-6}$$

Figure 2-1 depicts the use of the compound amount and present worth factors. In this diagram, capital is represented by the vertical axis and time by the horizontal axis. The direction of flow of capital is represented by the direction of the arrow; an upward pointing arrow represents either an outflow or inflow of capital where as the downward pointing arrow represents the opposite flow.

Uniform Compound Amount Factor

If a series of uniform receipts or dispersments, R, are made at the end of each year, for n years; a factor that will give the future value, S, of this series can be derived as follows:

Referring to Figure 2-2,

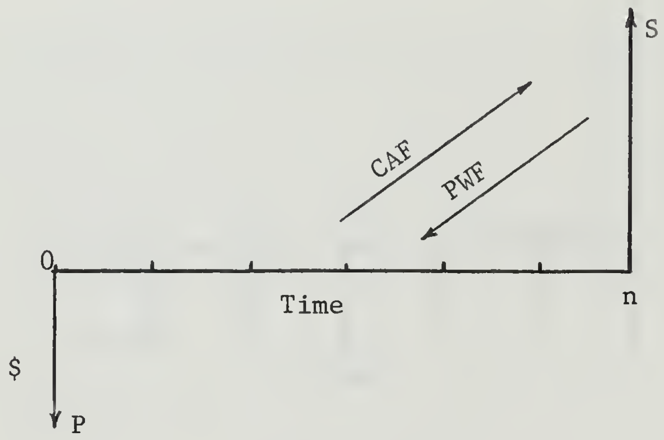

Figure 2-1

Use of the Compound Amount and Present Worth Factors[8]

$$S = R + (R)CAF(i,1) + (R)CAF(i,2) + \ldots +$$
$$(R)CAF(i,n-2) + (R)CAF(i,n-1)$$

$$S = R\left[1 + (1+i) + (1+i)^2 + \ldots + (1+i)^{n-2} + (1+i)^{n-1}\right]$$

Multiplying by (1+i)

$$S(1+i) = R\left[(1+i) + (1+i)^2 + \ldots + (1+i)^{n-1} + (1+i)^n\right]$$

Subtracting

$$S(1+i) - S = R\left[-1 + (1+i)^n\right]$$

$$S = R\left[\frac{(1+i)^n - 1}{i}\right] \qquad (2\text{-}7)$$

The uniform compound amount factor is designated UCAF(i,n). This factor is used to express the equivalence between a uniform series of receipts or dispersments and a future sum at interest rate i, for n years.

$$UCAF(i,n) = \frac{(1+i)^n - 1}{i} \qquad (2\text{-}8)$$

$$S = (R)UCAF(i,n) \qquad (2\text{-}9)$$

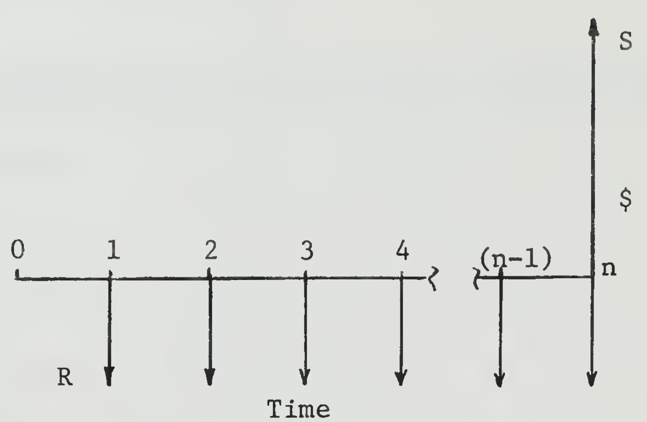

Figure 2-2
Uniform Series of Payments

For example, assume \$1,000 is placed in a savings account at the end of each year for five years, at 5% interest. The amount received at the end of five years would be:

$$S = \$1,000 \left[\frac{(1+.05)^5 - 1}{.05} \right]$$

$$= \$1,000(5.526)$$

$$= \$5,526$$

Sinking Fund Factor

Solving equation (2-7) for R yields:

$$R = S \left[\frac{i}{(1+i)^n - 1} \right] \qquad (2\text{-}10)$$

The quantity $i/((1+i)^n - 1)$ is known as the sinking fund factor, and is designated SFF(i,n). This factor expresses the equivalence between a future sum, S, and a uniform series of payments, R, at rate i, for n years.

$$SFF(i,n) = \frac{n}{(1+i)^n - 1} \qquad (2\text{-}11)$$

$$R = (S)SFF(i,n) \qquad (2\text{-}12)$$

Figure 2-3 depicts the use of the uniform compound amount and the sinking fund factors.

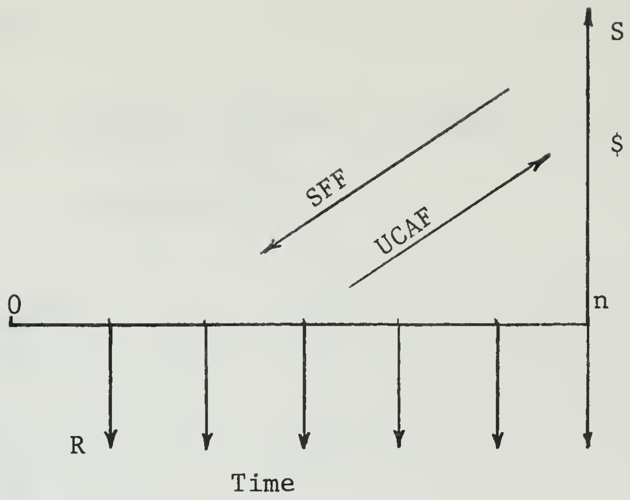

Figure 2-3

Use of the Uniform Compound Amount and Sinking Fund Factors[8]

Capital Recovery Factor

If a sum of money, P, borrowed at time zero, is to be repaid by a series of uniform payments, R, over n years, at interest rate i, then the factor that when multiplied by P will give R, is derived as follows[4]:

Referring to Figure 2-4,

$$P = (R)PWF(i,1) + (R)PWF(i,2) + (R)PWF(i,3) + \dots + (R)PWF(i,n-1) + (R)PWF(i,n)$$

$$P = R(1+i)^{-1} + R(1+i)^{-2} + \dots + R(1+i)^{-(n-1)} + R(1+i)^{-n}$$

$$P = R(1+i)^{-1}\left[1 + (1+i)^{-1} + \dots + (1+i)^{-(n-2)} + (1+i)^{-(n-1)}\right]$$

Multiplying by $(1+i)^{-1}$

$$P(1+i)^{-1} = R(1+i)^{-1}\left[(1+i)^{-1} + (1+i)^{-2} + \ldots + (1+i)^{-(n-1)} + (1+i)^{-(n)}\right]$$

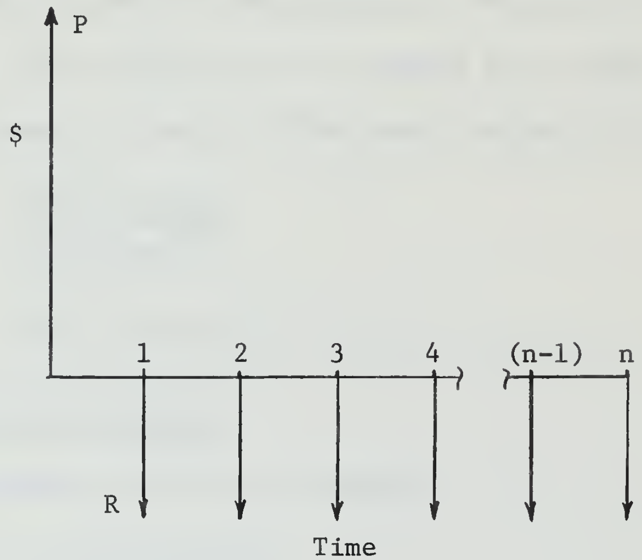

Figure 2-4
Uniform Series of Payments

Subtracting

$$P(1+i)^{-1} - P = R(1+i)^{-1}(-1 + (1+i)^{-n})$$

$$R = P\left[\frac{(1+i) - 1}{1 - (1+i)^{-n}}\right]$$

$$R = P\left[\frac{i}{1 - (1+i)^{-n}}\right] \tag{2-13}$$

The factor $i/(1 - (1+i)^{-n})$ is known as the capital recovery factor; and it is designated CRF(i,n). This factor expresses the equivalence between a present sum, P, and a series of equal payments, R, at interest

rate i, for n years.

$$\mathrm{CRF}(i,n) = \frac{i}{1 - (1+i)^{-n}} \tag{2-14}$$

$$R = (P)\mathrm{CRF}(i,n) \tag{2-15}$$

As an example, consider you purchase a house with a \$20,000 twenty-year mortgage. The mortgage is to be repaid at a uniform rate every year at an interest rate of 8%. The annual payments would then be:

$$R = \frac{\$20{,}000(.08)}{1-(1+.08)^{-20}}$$

$$R = \$2{,}037 \text{ per year}$$

Uniform Present Worth Factor

Solving equation (2-13) for P yields:

$$P = R\,\frac{(1 - (1+i)^{-n})}{i} \tag{2-16}$$

The resulting quantity $(1 - (1+i)^{-n})/i$ is known as the uniform present worth factor; and it is designated UPWF(i,n). This factor expresses the equivalence between future uniform payments, R, of a series, and a present amount, P, at interest rate i, for n years.

$$\mathrm{UPWF}(i,n) = \frac{1 - (1+i)^{-n}}{i} \tag{2-17}$$

$$P = (R)\mathrm{UPWF}(i,n) \tag{2-18}$$

Figure 2-5 depicts the use of the capital recovery and uniform present worth factors.

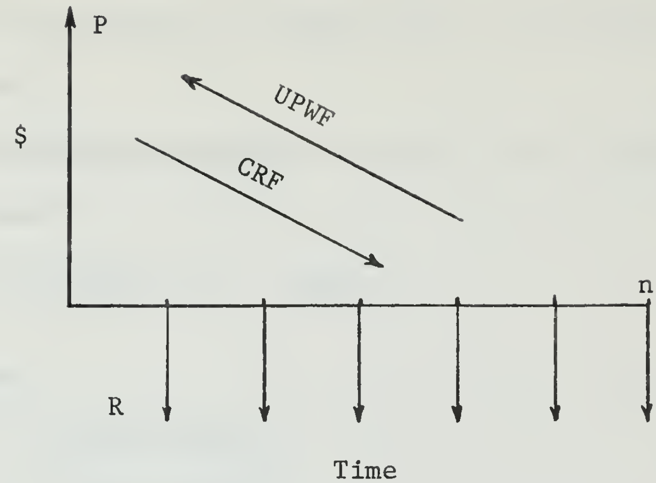

Figure 2-5
Use of the Capital Recovery and Uniform Present Worth Factors[8]

If time periods other than a year are to be used, both the interest rate and the time period must be for the same unit of time. That is, if compounding is to be on a monthly basis, interest, i, must be a monthly interest rate, and n, the number of time periods, must be in months.

For example, if $1,000 were compounded semiannually at 5% for six years: P = $1,000, n = 12, i = 2.5%

$$
\begin{aligned}
S &= (P)CAF(.025,12) \\
&= \$1,000(1.344) \\
&= \$1,344
\end{aligned}
$$

Another method of handling other than yearly periods is to determine an effective annual interest and make all computations on a yearly basis. The effective yearly interest may be obtained from[3]:

$$(1 + i/c)^c - 1$$

where i is the stated yearly interest and c is the number of compounding periods per year.

Using the previous example: $i = 5\%$, $c = 2$. The effective annual interest is then:

$$(1 + .05/2)^2 - 1 = .0506$$

Then S would be

$$\begin{aligned} S &= (P)CAF(.0506,6) \\ &= \$1,000(1.0506)^6 \\ &= \$1,344 \end{aligned}$$

III. DEPRECIATION

Physical assets, with the exception of land, generally lessen in value with use and the passage of time. They must eventually be replaced. Accounting for this reduction in value is known as depreciation.

Capital assets are purchased with the belief that they will earn more revenue over their life than they cost. This revenue results from the sale of products or services produced by the asset. A portion of anticipated revenue is considered to be capital recovered. Capital recovered should equal the initial cost of the asset less its salvage value at the time of retirement. Another part of revenue earned is considered to be a return on investment. Capital invested in an asset is recovered in small increments over its life. At any time, the interest on the unreturned investment must be considered a cost of ownership. It is an opportunity cost because the capital could have been used to earn a return in some other investment. The interest on the unreturned balance should equal the opportunity foregone in purchasing the asset. An investment in an asset is expected to return not only the capital invested, but also provide for interest earnings on the unreturned capital[3].

Depreciation is part of the cost of production; it is the cost of investment recovery plus return. To determine this cost it is necessary to predict the future value of the asset. This is accomplished by the selection of one of several mathematical models, which will be discussed below. These models assume a yearly decrease in the value of the asset.

Depreciation is complicated because it is not always possible to select a method of accounting for the lessening in value of the asset that will equal the actual lessening in value as a result of use and the passage of time, for this accounting requires a prediction of the future.

However, from the engineer's viewpoint, it makes no difference what depreciation model is used for investment recovery over time; for the present worth of the recovery plus return for all of the models is equal[5]. This fact will be demonstrated shortly. It is only necessary that the prediction of service life and salvage value be correct, and this is the engineer's problem. Thus the use of the present worth or capital recovery factor for selecting alternative capital investments will result in the same conclusion no matter what depreciation model is chosen.

When using depreciation for tax purposes, the model selected does make a difference; and this is discussed in Chapter IV.

Straight-Line Depreciation

The most basic and common form of depreciation model is the straight-line method. This model assumes that the value of an asset decreases at a constant rate. With the capital recovered each year given by:

$$\frac{P - S}{n} \tag{3-1}$$

where

n = Life of the asset in years

P = Initial capital investment

S = Salvage value year n

x = Cost of capital

As an example of the straight-line method, consider that an asset costs $11,000, has an estimated life of five years, an estimated salvage value of $1,000, and the desired rate of return is 10%. The yearly depreciation, given by equation (3-1), would be $2,000 and the return on the investment is given in Table 3-1.

Table 3-1

Straight-Line Depreciation

Year k	Capital Unrecovered Beginning of Year k Z(k-1)	Capital Recovered A(k)	Return on Capital Unrecovered xZ(k-1)	Capital Recovered Plus Return A(k)+xZ(k-1)	Present Worth of A(k)+xZ(k-1) at Time k=0
1	$11,000	$ 2,000	$1,100	$3,100	$ 2,817.90
2	9,000	2,000	900	2,900	2,395.40
3	7,000	2,000	700	2,700	2,027.70
4	5,000	2,000	500	2,500	1,707.50
5	3,000	2,000	300	2,300	1,428.30
		$10,000			$10,376.80

Declining Balance Method

Under this method, a constant percentage of the remaining undepreciated value is recovered each year. The reduction in the unrecovered value of an asset decreases at a decreasing rate. The depreciation during any year is equal to the undepreciated amount at the beginning

of that year times d; where d is the fixed percentage rate of depreciation. The undepreciated balance remaining at the end of any year equals the unrecovered balance at the beginning of the year times (1-d). The unrecovered capital at the end of n years is then $P(1-d)^n$. This then equals the salvage value of the asset:

$$S = P(1-d)^n$$

The value of d is then given by:

$$d = 1 - (S/P)^{1/n} \qquad (3\text{-}2)$$

Considering the example used under the straight-line method, d would equal .381; and the capital recovery plus return would be as given in Table 3-2.

Table 3-2

Declining Balance Depreciation

Year k	Capital Unrecovered Beginning of Year k Z(k-1)	Capital Recovered A(k) = dZ(k-1)	Return on Capital Unrecovered xZ(k-1)	Capital Recovered Plus Return A(k)+xZ(k-1)	Present Worth of A(k)+xZ(k-1) at time k=0
1	$11,000.00	$ 4,191.00	$1,100.00	$5,291.00	$ 4,809.52
2	6,809.00	2,594.22	680.90	3,275.12	2,705.24
3	4,214.78	1,605.83	421.48	2,027.31	1,522.51
4	2,608.95	994.01	260.90	1,254.91	857.10
5	1,614.94	615.29	161.49	776.78	482.38
		$10,000.35			$10,376.75

Sum-of-the-Years-Digit Method

This model also assumes that the value of an asset decreases at a decreasing rate. The amount recovered for any year, k, is found by:

$$A(k) \quad = \quad (P-S) \left[\frac{n - (k-1)}{\sum\limits_{n=1}^{n} n} \right] \qquad (3\text{-}3)$$

where A(k) = capital recovered during year k.

Considering the example used under the straight-line method, the capital recovery plys return using the sum-of-the-years-digit method is given in Table 3-3.

Table 3-3

Sum-of-the-Years-Digit Depreciation

Year k	Capital Unrecovered Beginning of Year k Z(k-1)	Capital Recovered A(k)	Return on Capital Unrecovered xZ(k-1)	Capital Recovered Plus Return A(k)+xZ(k-1)	Present Worth of A(k)+xZ(k-1) at Time k=0
1	$11,000.00	$ 3,333.33	$1,100.00	$4,433.33	$ 3,029.90
2	7,666.67	2,666.67	766.67	3,433.34	2,835.94
3	5,000.00	2,000.00	500.00	2,500.00	1,877.50
4	3,000.00	1,333.33	300.00	1,633.33	1,115.56
5	1,666.67	666.67	166.67	833.34	517.50
		$10,000.00			$10,376.40

Figure 3-1 depicts the three methods of depreciation using the examples of Tables 3-1, 3-2, and 3-3.

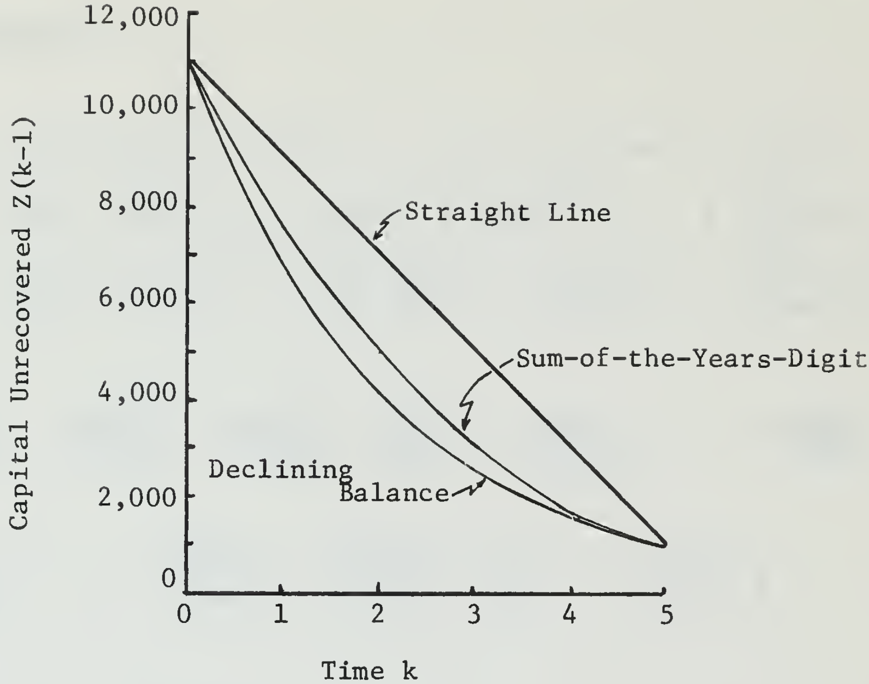

Figure 3-1

Methods of Depreciation

In general, for any method of depreciation:

$$P-S = \sum_{k=1}^{n} A(k)$$

and the interest for the unrecovered portion of the investment during year k equals:

$$x\left[S + \sum_{k=k}^{n} A(k) \right]$$

The present worth of the return plus capital recovery over the life of the asset is then:

$$\sum_{k=1}^{n} \left[A(k) + x(S + \sum_{k=k}^{n} A(k)\ ) \right] (1 + x)^{-k}$$

This can be expanded to:

$$\left[A(1) + x(S + A(1) + A(2) + \ldots + A(n)\ )\right](1+x)^{-1} + \left[A(2) + x(S + A(2) + A(3) + \ldots + A(n)\ )\right](1+x)^{-2} + \ldots + \left[A(n) + x(S + A(n)\ )\right](1+x)^{-n} + S\left[\frac{x}{(1+x)} + \frac{x}{(1+x)^2} + \ldots \frac{x}{(1+x)^n}\right]$$

$$\frac{A(1)}{(1+x)} + \frac{xA(1)}{(1+x)} + \frac{xA(2)}{(1+x)} + \frac{A(2)}{(1+x)} + \frac{xA(2)}{(1+x)^2} + \ldots + \frac{xA(n)}{(1+x)} + \frac{xA(n)}{(1+x)^2} + \ldots + \frac{xA(n)}{(1+x)^n} + \frac{A(n)}{(1+x)^n} + S\left[1 - \frac{1}{(1+x)^n}\right]$$

$$A(1)\frac{(1+x)}{(1+x)} + \frac{xA(2)}{(1+x)} + A(2)\frac{(1+x)}{(1+x)^2} + \ldots + A(n)\left[1 - \frac{1}{(1+x)^n}\right] + \frac{A(n)}{(1+x)^n} + S\left[1 - \frac{1}{(1+x)^n}\right]$$

$$A(1) + A(2)\frac{(1+x)}{(1+x)} + \ldots + A(n) + \frac{A(n)}{(1+x)^n} + S\left[1 - \frac{1}{(1+x)^n}\right]$$

which reduces to[5]:

$$\begin{aligned} & A(1) + A(2) + \ldots + A(n) + S\left[1 - \frac{1}{(1+x)^n}\right] \\ &= \sum_{k=1}^{n} A(k) + S - S(1+x)^{-n} \\ &= P - S(1+x)^{-n} \end{aligned}$$

This result shows that the present worth of the recovery cost plus return for any method of depreciation is equal to the initial investment minus the present worth of the salvage value. Tables 3-1,

3-2, and 3-3 demonstrate this fact. The small discrepancy in the present worth total is due to rounding error.

As long as the retirement of an asset takes place at the age and salvage value for which capital recovery and return were calculated, the sum of the present worth of the recovery plus return will be equivalent for any method of depreciation.

IV. TAXES

Taxes levied by federal, state, and local governments are a significant part of the cost of operating a business, and are therefore important in the analysis of engineering decisions. The laws governing the various taxes are rather complex, and except for federal income tax, vary with the location of the firm. The knowledge required to be expert in taxation has made it a field of study for lawyers and accountants. However, every manager who makes business decisions must possess some knowledge of taxes so that their implications are fully integrated into the decision process. Because of the variety of state and local taxes, only the federal corporate income tax will be discussed. However, a method for including state corporate income taxes into the engineering decision will be presented.

Most management decisions are choices between alternative courses of action. For example, should we finance capital expenditures through bank loans or the sale of stock; should we buy or lease a piece of equipment; what method of depreciation should we use? The proper choice of alternatives will lead to a lower income tax, and a higher profit. Therefore, the effect of income taxes must be included in the evaluation process to insure that the true cost of the alternatives is considered[5].

Federal Corporate Income Tax

Federal income taxes were established with the passage of the Sixteenth Amendment to the Constitution in 1913. Corporate income tax rates have been modified on numerous occasions by Congress. At present the tax is divided into two parts: on the first $25,000 of

taxable income the rate is 22%; on taxable income above \$25,000 the rate is 48%. In computing the tax it is convenient to multiply the taxable income by 48% and then subtract \$6,500, which is (.48-.22) (\$25,000). The tax on a taxable income of \$50,000 would be:

Method 1	(\$25,000)(.22) + (\$25,000)(.48)	=	\$17,500
Method 2	(\$50,000)(.48) - \$6,500	=	\$17,500

Project Versus Average Tax Rate

When analyzing tax expenses it is important to distinguish between the project tax rate and the average tax rate. The average tax rate for a corporation with a taxable income, including the income from the project, of \$50,000 would be:

$$\text{Average tax rate} = \frac{\text{Tax}}{\text{Taxable income}}$$

$$= \frac{\$17{,}500}{\$50{,}000}$$

$$= 35\%$$

However, if the taxable income without the project is \$40,000, and the taxable income contributed by the project is \$10,000, then the project tax rate is computed as follows:

$$\text{Project tax rate} = \frac{\text{Project tax}}{\text{Project taxable income}}$$

$$\text{Project tax} = \text{Tax with the project} - \text{Tax without the project}$$

$$\text{Project tax} = \$17{,}500 - \$12{,}700$$

$$= \$4{,}800$$

$$\text{Project tax rate} = \frac{\$\ 4,800}{\$10,000}$$

$$= 48\%$$

Thus in this case, the project tax rate is the marginal tax rate; that is, the tax rate on the last dollar of income from the project. If the taxable income of a firm, without the taxable income from the project, is above \$25,000, then the marginal tax rate will equal the project tax rate. If the taxable income both with and without the project is below \$25,000, then the tax rate will be 22%. If the project increases the taxable income from below to above \$25,000, the project tax rate will be the average tax rate for the income derived from the project. For this paper, it will be assumed that the taxable income of the firm before considering the project is above \$25,000, and thus the project tax rate will equal the marginal tax rate of 48%[5].

Taxable Income

For a corporation taxable income is obtained as follows[6]:

Income broadly conceived - Exclusions = Gross Income

Gross Income - Routine Deductions - Special Deductions = Taxable Income

where

Exclusions	= Interest from state and local government debt obligations; recovery of prior taxes; and life insurance proceeds received at corporate officer's death.
Gross Income	= Gross revenue less cost of goods sold; revenue from services rendered; dividends, interest, rents and royalties received; and gain on the sale of property.
Routine Deductions	= Wages; salaries; rents; repairs; bad debts; interest; taxes; contributions; depreciation;

depletion; advertising; research and development; losses on sale of property; and other operating expenses.

Special Deductions = A deduction equal to 85% of the dividend received from domestic corporations; an optional amortization of organizational expenses.

For most engineers who are evaluating alternative projects, the above general formula for taxable income can be simplified to:

Project Gross Income = Gross revenue less cost of goods sold; revenue from services rendered; and gain on the sale of property.

Routine Deductions = Wages; salaries; rents; repairs; interest; taxes; research and development; other operating expenses; and depreciation.

Depreciation

The federal corporate income tax laws permit the depreciation of physical assets, except land and inventories, to be included as a deduction in determining taxable income. The method of depreciation will be shown to be important in the determination of taxable income.

When computing depreciation it is necessary to establish the useful life and salvage value of the asset, and the method of depreciation. For tax purposes it is an advantage to select the shortest asset life and the highest rate of depreciation allowed by the Internal Revenue Service. Doing so will minimize the present worth of taxes paid, although the total tax will be the same. It is possible that the asset's life selected for tax purposes will be different from the actual service life.

For example, using the three methods of depreciation discussed in Chapter III, an asset with a cost of $11,000; a salvage value of $1,000;

a useful life of five years; a taxable income, before the deduction for depreciation of $5,000; a marginal tax rate of 48%; and an effective cost of capital of 10%; the tax outlay would be:

a. Straight-Line Method. Table 4-1 shows the yearly tax and the present worth of the taxes using the straight-line method of depreciation.

Table 4-1

Income Tax Using the Straight-Line Method of Depreciation

Year	Depreciation	Taxable Income	Tax	Present Worth of Tax
1	$2,000	$3,000	$1,440	$1,308.96
2	2,000	3,000	1,440	1,189.44
3	2,000	3,000	1,440	1,081.44
4	2,000	3,000	1,440	983.52
5	2,000	3,000	1,440	894.24
Total			$7,200	$5,457.60

b. Declining Balance Method. The tax law limits the maximum percentage allowable for the declining balance method to no more than twice the straight-line rate. For the example used above, the straight-line rate is 20%. The declining balance rate is then limited to 40%. For this example the rate of 38.1%, which was obtained in Chapter III in order to have a salvage value of $1,000, will be used. Table 4-2 presents the tax and the present worth of the tax using this method.

Table 4-2

Income Tax Using the Declining Balance Method of Depreciation

Year	Depreciation	Taxable Income	Tax	Present Worth of Tax
1	$4,191.00	$ 809.00	$ 388.32	$ 352.98
2	2,594.22	2,405.78	1,154.79	953.86
3	1,605.83	3,394.17	1,629.31	1,223.61
4	994.01	4,005.99	1,922.90	1,313.34
5	615.29	4,384.71	2,104.68	1,307.01
Total			$7,200.00	$5,150.80

c. Sum-of-the-Years-Digit Method. Again using the same example, Table 4-3 presents the tax and present worth of the tax using the sum-of-the-years-digit method of depreciation.

Table 4-3

Income Tax Using Sum-of-the-Years-Digit Method of Depreciation

Year	Depreciation	Taxable Income	Tax	Present Worth of Tax
1	$3,333.33	$1,666.67	$ 800.00	$ 727.20
2	2,666.67	2,333.33	1,120.00	925.12
3	2,000.00	3,000.00	1,440.00	1,081.44
4	1,333.33	3,666.67	1,760.00	1,202.08
5	666.67	4,333.33	2,080.00	1,291.68
Total			$7,200.00	$5,227.52

As demonstrated by the above examples, the total tax paid over the life of the asset is independent of the method of depreciation; however, the present worth of the tax is a minimum for the declining balance method[5].

The depreciation method selected for use in this paper is the straight-line method because:

a. Many firms use this method for tax purposes.

b. For the analysis of alternative projects this method is commonly used in industry even though one of the other methods might be used by the firm for accounting and tax purposes.

c. This method is the most conservative, giving the highest present worth of tax expenditure.

d. The accelerated methods have come under attack from various sources and they could be eliminated by Congress.

Interest

Interest paid on indebtedness is considered a deductible expense under present tax laws. The method employed to finance a project determines the taxes to be paid and the total cost of the project.

For example, assume an asset costing $10,000 is financed through a bank loan at an interest rate of 10% on the unpaid balance; the principal to be repaid in $2,000 yearly increments over five years. Assume the taxable income, excluding the interest deduction, is $3,000. Then the taxes paid each year and their present worth are given in Table 4-4. It will be noted that the present worth of taxes is the same for each year. This is because the interest rate used to compute the present worth is the same rate used to compute the interest each year.

Table 4-4

Effect of Debt Financing on Taxes

Year	Outstanding Debt	Interest	Taxable Income	Taxes	Present Worth of Taxes
1	$10,000	$1,000	$2,000	$ 960	$ 872.64
2	8,000	800	2,200	1,056	872.64
3	6,000	600	2,400	1,152	872.64
4	4,000	400	2,600	1,248	872.64
5	2,000	200	2,800	1,344	872.64
Total				$5,760	$4,363.20

If the asset were to be purchased with equity capital, that is by using retained earnings or through the sale of stock, the taxes and present worth of taxes would be as given in Table 4-5.

The taxes paid each year and the present worth of taxes paid are larger when equity capital is used to finance the asset. This is not to imply that debt financing will always be the method selected, for there are other considerations to be included, such as: the need for liquidity of capital, the total amount of debt permitted by the government if the industry is regulated, the amount of debt financing available, and the relative magnitude between the effective interest rate obtainable and the equity return that must be made to the stockholders in order to induce them to make the investment.

Table 4-5

Effect of Equity Financing on Taxes

Year	Interest	Taxable Income	Taxes	Present Worth of Taxes
1	0	$3,000	$1,440	$1,308.96
2	0	3,000	1,440	1,189.44
3	0	3,000	1,440	1,081.44
4	0	3,000	1,440	983.52
5	0	3,000	1,440	894.24
Total			$7,200	$5,457.60

The reduction in taxes, due to the deductible nature of interest expenses, effectively reduces the interest rate on the loan[5]. In the examples presented in Tables 4-4 and 4-5, the tax during the first year was $960 with debt financing and $1,440 with equity financing. The savings in taxes is then $1,440 minus $960, or $480. This savings can be considered to reduce the interest payment from $1,000 to $520. The effective interest rate is then $520/$10,000 or 5.2%. This same result would be obtained for each succeeding year as demonstrated in Table 4-6.

The effective interest rate can be computed using:

$$EI = i - ir$$

where

EI = Effective interest rate

i = Bank or bond interest rate

r = Effective income tax rate

Table 4-6

Effective Interest Rate

Year	Outstanding Debt	Tax Debt	Tax Equity	Savings On Taxes	Interest Payments	Effective Interest	Effective Interest Rate
1	$10,000	$ 960	$1,440	$480	$1,000	$520	5.2%
2	8,000	1,056	1,440	384	800	416	5.2
3	6,000	1,152	1,440	288	600	312	5.2
4	4,000	1,248	1,440	192	400	208	5.2
5	2,000	1,344	1,440	96	200	104	5.2

Using the above example: i = 10%, r = .48, then EI = 5.2%.

Two methods have been presented for taking into consideration the deductible nature of interest payments on taxes; the accounting method, where the interest payment is included as a deduction in determining taxable income; and the effective interest method, where the interest rate is reduced by that amount attributable to the savings in taxes when using debt financing. The latter method will be employed in this paper.

Effective Corporate Income Tax Rate

There are various state and local taxes which could be taken into account when selecting alternative projects for investment. These include the state corporate stock tax, state gross receipts tax, sales tax, and real estate tax. These taxes may, or may not affect the outcome of the selection process. In any case, they vary considerably from location to location and cannot be considered in a general model. However, state corporate income taxes are more consistent and can be

included in a general model, and a method of handling them will be considered. The state corporate income tax for Pennsylvania will be used to demonstrate how this tax can be taken into consideration.

The effective income tax rate is defined as that tax rate that integrates the federal and state corporate income taxes into one rate, and it is derived as follows[7]:

FIT = Federal income tax

SIT = State income tax

TIF = Taxable income federal

TIS = Taxable income state

TIFP = Taxable income federal for a project

r = Effective income tax rate

$$\text{FIT} = .48(\text{TIF}) - \$6{,}500$$

$$\text{SIT} = .12(\text{TIS})$$

$$\text{TIS} = \text{TIF} + \text{SIT}$$

$$\text{SIT} = .12(\text{TIF} + \text{SIT})$$

$$\text{SIT} = .136364(\text{TIF})$$

$$\text{Total Income Tax} = \text{FIT} + \text{SIT}$$

$$= .616364(\text{TIF}) - \$6{,}500$$

If the firm's taxable income before the new project is considered is more than \$25,000, then the total income tax for the project would be:

$$.616364(\text{TIFP}) \tag{4-3}$$

The effective income tax rate would be:

$$r = .616364 \tag{4-4}$$

V. COST OF CAPITAL

In the preceding chapters, the interest rate or cost of capital was assumed to be known; in this chapter, a procedure for estimating the cost of capital will be discussed. If projects are selected using the concept of the time value of money, the cost of capital must be used in the process. Accurate prediction of the cost of capital is therefore required for the proper selection of alternative projects.

An approach to measuring the cost of capital is to combine the cost of the separate sources into a weighted average. The problem then is to determine the cost of capital for each source. Capital can be obtained through either equity or debt financing, or a combination of the two. Equity financing is obtaining capital through the sale of stock and use of retained earnings. Debt financing consists of obtaining loans from banks or other credit institutions, and through the sale of bonds.

Common Stock and Retained Earnings

From the point of view of the present common stockholders, any new project which would produce a lower rate of return than they are currently enjoying will operate to their disadvantage. Therefore, to sell a new issue of common stock in order to finance a new project, the rate of return whould be no lower than currently exists. The cost of the common is then[1]:

$$\text{Cost of Common} = \frac{\text{Current earnings per share of common}}{\text{Proceeds per share}} \tag{5-1}$$

where

Current earnings per share of common = Current dividends per share + retained earnings per share

Proceeds per share = Market price per share - selling cost per share - discount

The proceeds to the company per share of common sold is usually lower than the market price because the company must pay investment banking fees. Also, the shares might have to be sold at a bargain price in order to stimulate interest in the new issue.

By employing the above procedure, retained earnings are included in the common stock component. The market price of the common stock includes the expectation of the stockholders that a firm will earn profits, distribute some of them as dividends, and retain some for future use. Thus the current market price reflects the stockholders' beliefs concerning the future of the firm which necessarily includes consideration of retained earnings in the cost of the common.

Preferred Stock

The current market yield will be used as the cost of this source[1]:

$$\text{Cost of preferred} = \frac{\text{Current annual dividends per share of preferred}}{\text{Current market price}} \tag{5-2}$$

Bonds

The cost of bonds to the company is given by[1]:

$$\text{Cost of bonds} = \frac{\text{Average yearly payments}}{\text{Average proceeds}} \tag{5-3}$$

The rate of interest stated on the bond is known as the face rate. This rate may not be the cost of the bond to the company. The bonds may be sold at a discount or a premium. In addition, the proceeds received by the company may be reduced by the cost of selling the bond. The following examples will demonstrate how to compute the cost of the bonds to the company:

Assume the face rate of a twenty-year, $1,000 bond is 5%. The company receives $1,000 for the bond and the selling cost is negligible. The average yearly payment is then $50 and the cost of the bond is 5% or the face rate.

Now assume the bond was sold to an investment banker for $1,040, and the banker sold the bond to the public for $1,045. The company receives a premium of $40. During the life of the bond the company will pay $50 a year in interest, and at maturity will return the $1,000 principal. The premium, when averaged over the twenty-year life of the bond, is $2 per year. This $2 can be considered to reduce the yearly interest payment to $48. The average proceeds would be the average of the amount received and the amount returned to the bond holder at maturity. The cost of the bond is then:

$$\text{Cost of bond} = \frac{\$48}{1/2(\$1{,}000+1{,}040)}$$

$$= 4.7\%$$

If the bond was sold to an investment banker for $960, the company sold at a discount of $40. Since the company must return $1,000 to the bondholder at maturity, this $40 discount is an average yearly loss to the company of $2. The average interest cost can be

considered to be the stated value of $50 plus the $2 per year loss. The cost of the bond is then:

$$\text{Cost of bond} = \frac{\$52}{1/2(\$1{,}000+\$960)}$$

$$= 5.3\%$$

To sell bonds a company must pay the going rate of interest. Selling the bond at a premium or discount allows the company to make adjustments in the yield, without changing the face rate of interest which would necessitate a new bond issue. The rate of interest necessary to attract investors varies from company to company depending on the investors' appraisal of the risk involved in the investment.

Bank Loan

For long term bank loans, one common method of repayment is for the borrower to make installment payments on the loan, with the interest being paid only on the unpaid balance. In this situation the true rate of interest equals the quoted rate.

Bank loans are also made where the interest and principal are repaid in a lump sum, or where the interest is deducted from the principal at the time of the loan, or the loan may be paid in installments with the interest charge being made on the original principal each period. The true cost of the loan can be obtained by use of the following equation[1]:

$$\text{Cost of loan} = \frac{2cf}{P(n+1)}$$

where

c = Number of equal installment payments in one year

f = Total interest and finance charges in dollars

n = Total number of installment payments to be made

P = Amount of cash actually received

For example, assume a firm borrows $1,000 at 5% for one year, the interest to be deducted at the time of the loan.

$c = 1$, $f = \$50$, $P = \$950$, $n = 1$

$$\text{Cost of loan} = \frac{2(50)}{950(2)}$$

$$= 5.26\%$$

If the interest were to be paid at maturity: $P = \$1{,}000$

$$\text{Cost of loan} = \frac{2(50)}{1000(2)}$$

$$= 5\%$$

If the loan was to be repaid in twelve monthly installments:

$c = 12$, $f = \$50$, $P = \$1{,}000$, $n = 12$

$$\text{Cost of loan} = \frac{(2)(12)(50)}{(1000)(13)}$$

$$= 9.23\%$$

Effective Cost of Capital

Now that a value has been obtained for the percentage cost of each source of capital, it is necessary to obtain a weighted average cost of capital. This will be accomplished by first obtaining a weighted average cost of equity capital and of debt capital. The weighted average of these two values will then give the weighted

average cost of capital.

The weighted average cost of equity capital is obtained by taking the ratio of the capital supplied by each source to the total equity capital, and multiplying by the percentage cost of capital for that source. The sum of those products is the weighted average cost of equity capital. In a similar fashion the weighted average cost of debt capital is obtained. Table 5-1 demonstrates this procedure.

The weighted average cost of capital is then obtained from[2]:

$$a = ib + j(1-b)$$

where

a = Weighted average cost of capital

b = Debt to total capital ratio

i = Weighted average cost of debt capital

j = Weighted average cost of equity capital

Table 5-1

Weighted Cost of Equity and Debt Capital

Source	Capital Supplied	% of Total	% Cost of Capital	Weighted Cost %
Equity capital				
Common stock	$ 90,000	90	10	9.0
Preferred stock	10,000	10	5	.5
Total Equity Capital	$100,000			9.5%
Debt capital				
Bank loan	$ 30,000	60	6	3.6
Bonds	20,000	40	5	2.0
Total Debt Capital	$ 50,000			5.6%

Continuing with the example of Table 5-1:

$i = 5.6\%$, $j = 9.5\%$, $b = \$50{,}000/\$150{,}000 = .333$

$$a = (.056)(.333) + (.095)(1 - .333)$$
$$= .0821$$

As discussed in Chapter IV, the savings in income tax, due to the deductible nature of the interest paid on debt capital, effectively reduces the interest rate of debt financing. The effective cost of capital can therefore be defined as follows[2]:

$$x = ib + j(1-b) - ibr \qquad (5\text{-}4)$$

where

x = Effective cost of capital

r = Effective income tax rate

With $r = .61$, the effective cost of capital for the preceding example would be:

$$x = .0821 - (.056)(.333)(.61)$$
$$= 7.07\%$$

VI. DECISION MODEL

There exists a number of different criteria that can be used in selecting alternative projects for investment. Some criteria include the time value of money, others do not. It is apparent from our previous discussion that consideration must be given to the time value of money in order to insure an accurate economic analysis.

Generally the basic criteria used in selecting between alternative investment opportunities with all other considerations being equal, is to select the one that is most profitable. In determining the profit that would be produced by alternative projects, it would be necessary to predict the cost and revenue attributable to the projects over their lives. This would require such assumptions as the future state of the economy; the amount of inflation; and the future demand for the product. The fewer the assumptions that must be made, the more accurate will be the resulting decision. One method of reducing the assumptions is to base the selection on cost rather than on profit. If the alternative projects can be expected to produce equal revenue over their life, the one that resulted in the minimum cost over its life would be the one that would produce the maximum profit over its life.

Present Worth of Cost

Two criteria commonly used are the present worth of cost and the average annual cost. To employ the present worth of cost it is necessary that the projects have equal lives, otherwise erroneous conclusions would be reached. The average annual cost criteria allows comparison when the projects have unequal lives.

The following abbreviations will be used in this chapter:

AAC	=	Effective average annual cost
C	=	The cost of the asset at time zero
c(t)	=	Production cost, time t
CAF(x,t)	=	Compound amount factor, equation (2-1)
CRF(x,t)	=	Capital recovery factor, equation (2-14)
CRFS(x,t)	=	Capital recovery factor including salvage
CRFST(x,t)	=	Capital recovery factor including taxes and salvage
CRFT(x,t)	=	Capital recovery factor including taxes
E	=	Sum of present worth of operating expenses, time zero
e(t)	=	Expenses incurred in operating the asset, year t
m	=	Time required to produce the asset, in years
n	=	Life of the asset, in years
P	=	Sum of the present worth of expenses plus asset cost, time zero
PWF(x,t)	=	Present worth factor, equation (2-5)
R	=	Average annual cost if there were no taxes, or average annual net income required after deducting taxes
RT	=	Average annual cost including taxes, or average annual net income required before deducting taxes
S	=	Salvage value, year n

t = Time, year in question

x = Effective cost of capital, equation (5-4)

The present worth of cost of a project at time zero would be:

$$P = C + E \tag{6-1}$$

$$E = \sum_{t=1}^{n} e(t)PWF(x,t) \tag{6-2}$$

where

C = The cost of the asset at time zero

E = Sum of present worth of operating expenses, time zero

$e(t)$ = Expenses incurred in operating the asset, year t

It is assumed that the operating costs, e(t), are lump sum expenses occurring at the end of the year in question. These expenses include: wages and salaries, fringe benefits, insurance, fuel cost, maintenance and repair cost, spare parts, and any other cost incurred as a result of operating the project.

Time zero is defined as the time that the project first starts to perform its function productively.

The cost of the asset, C, is the cost at time zero. If the asset was not paid for in a lump sum at time zero, but over a period of time preceding time zero, then the actual cost of the asset must be computed by taking into account the time value of money. The cost would then be:

$$C = \sum_{t=1}^{m} c(t)CAF(x,t) \qquad (6\text{-}3)$$

where

$c(t)$ = Production cost, time t

For example, assume that a reactor core is being manufactured for a nuclear power plant. The core will take eighteen months to produce, and payments are required at various stages of the manufacturing process, as shown on the time-money diagram, Figure 6-1.

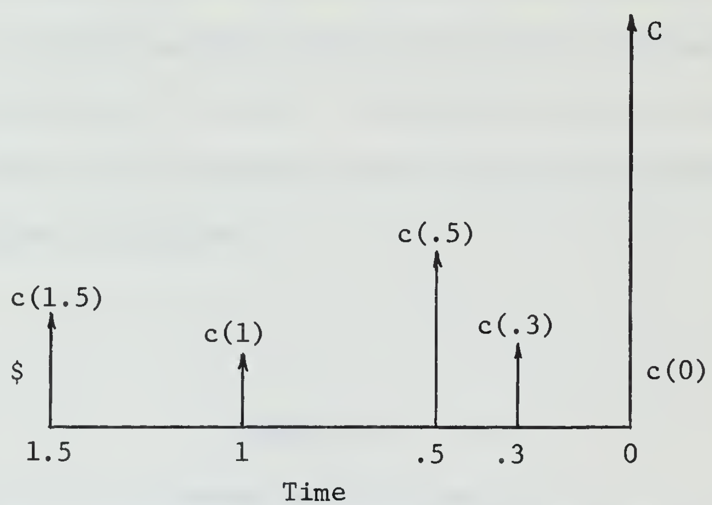

Figure 6-1

Time-Money Diagram

$$C = c(1.5)CAF(x,1.5) + c(1)CAF(x,1) + c(.5)CAF(x,.5) + c(.3)CAF(x,.3) + c(0)CAF(x,0)$$

Average Annual Cost

The average annual cost criteria carries the present worth of cost method one step further. The present worth of cost is simply multiplied by the capital recovery factor, equation (2-14), to give a uniform annual cost over the life of the asset:

$$R = (P)CRF(x,n) \tag{6-4}$$

This criteria is more general than the present worth criteria for the life of the alternative assets need not be equal.

As an example, consider that a selection is to be made between two machines. Machine A has a useful life of three years, a cost at time zero of $1,000; machine B has a useful life of four years, and a cost at time zero of $1,100. The effective cost of money for the company is 10%. The operating costs, e(t), and the present worth of costs are given in Table 6-1.

Table 6-1

Present Worth of Cost

		Machine A		Machine B	
Year	Designation	Cost	Present Worth of Cost	Cost	Present Worth of Cost
0	C	$1,000	$1,000.00	$1,100	$1,100.00
1	e(1)	100	90.90	100	90.90
2	e(2)	100	82.65	100	82.65
3	e(3)	100	75.13	100	75.13
4	e(4)			100	68.30
			$1,248.68		$1,416.98

The average annual cost is then computed using equation (6-4):

Machine A $\quad R = (\$1,248.68)CRF(.1,3)$

$= (\$1,248.68)(.402)$

$= \$501.97$

Machine B $\quad R = (\$1,416.98)CRF(.1,4)$

$= (\$1,416.98)(.315)$

$= \$446.34$

Thus, under the present worth of cost criteria, Machine A would have erroneously been selected as the one with the minimum cost. The average annual cost calculation shows that in reality Machine B has the lowest cost.

The average annual cost criteria assumes that the alternative projects are equal in all respects except for their useful life and cost. If this is not true, then an attempt must be made to express other differences in terms of cost; or to bring into the decision some other judgment process. Some examples of items which should be considered, but are not generally thought of as costs:

a. If the total down time, because of maintenance or some other reason, for one of the alternative projects was longer than the other, the income lost during the extra down time can be considered a cost for that project. For example, when comparing nuclear and fossil fueled power plants as alternative investments, the extra down time experienced by the nuclear power plant during refueling must be considered a cost to the nuclear power plant. This cost can be estimated by determining the income lost as a result of the plant not operating. This income would be the revenue that would have

been generated minus the operating expenses.

b. If one of the alternative projects is to have a greater output capacity than the other, and the firm cannot utilize the larger capacity, then the excess capacity is of no value and can be neglected. If, however, the firm can utilize the excess capacity, the projects can be compared on the basis of cost per unit of output.

Now that a method for selecting among alternative investments has been decided upon, it is necessary to expand the method to include the effect of a salvage value and of taxes on the alternative projects.

<u>Average Annual Cost With Salvage Value</u>

Many projects will have some salvage value at the end of their useful life. This value must then be included in the average annual cost decision model. This is accomplished by modifying the capital recovery factor as follows[8]:

Performing the same sequence of steps as done in the derivation of equation (2-13):

$$P = (R)PWF(x,1) + (R)PWF(x,2) + \ldots + (R)PWF(x,n) + (S)PWF(x,n)$$

$$P = (R)(1+x)^{-1} + (R)(1+x)^{-2} + \ldots + (R)(1+x)^{-n} + (S)(1+x)^{-n}$$

$$P = R\left[\frac{1 - (1+x)^{-n}}{x}\right] + S(1+x)^{-n}$$

$$R = \frac{(P-S)x}{1 - (1+x)^{n}} + xS$$

$$R = (P-S)CRF(x,n) + xS$$

$$\frac{R}{P-S} = CRF(x,n) + \frac{xS}{P-S}$$

$$CRFS(x,n) = CRF(x,n) + \frac{xS}{P-S} \quad (6\text{-}5)$$

$$R = (P-S)CRFS(x,n) \quad (6\text{-}6)$$

Average Annual Cost Including Taxes

As discussed in Chapter IV, taxes affect the cost of various projects and therefore must be included in the decision process. This is accomplished by modifying the capital recovery factor to account for the added cost of income taxes so that the resulting average annual cost is large enough to include all of the costs included in R, plus income taxes.

For a project to be accepted it must provide sufficient income to cover depreciation, all other costs associated with the asset, and an adequate return on the investment. The return on the investment is included by use of the equity return rate, j, in the derivation of the effective cost of capital, equation (5-4). Therefore, the average annual cost can also be thought of as the required average annual gross income. Part of this gross income is taxable, as depicted in Figure 6-2.

The present worth of the cost of the asset at time zero, C, is used in this derivation instead of the present worth of the operating cost plus asset cost, P, because only the cost of the asset is depreciable. The average annual operating cost will be included in the effective average annual cost, equation (6-11).

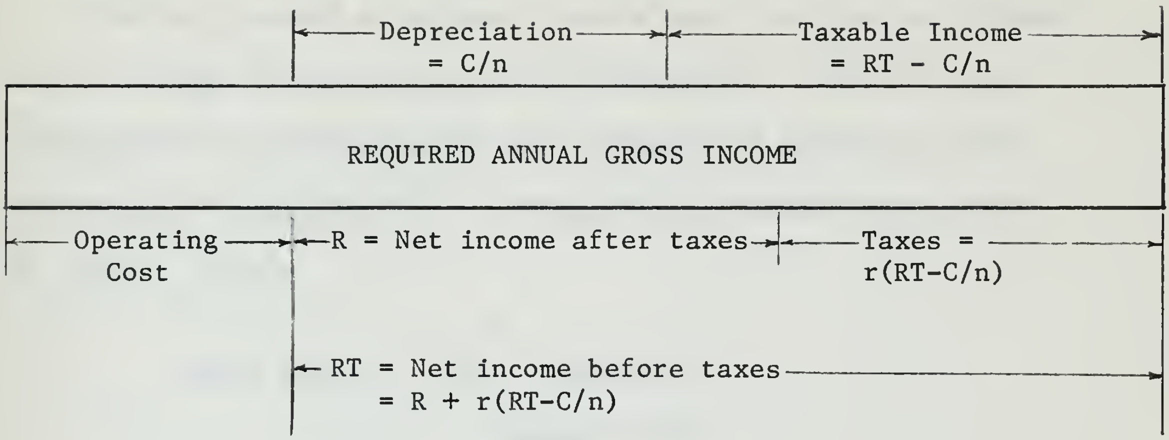

Figure 6-2

Division of Revenue[8]

As proposed in Chapter IV, the straight-line method of depreciation will be used for tax purposes, and will equal C/n. The derivation is then[8]:

$$\text{Annual Taxes} = r(RT - C/n)$$

$$RT = R + \text{Annual Taxes}$$

$$RT = R + r(RT - C/n)$$

$$RT = \frac{R - rC/n}{1 - r}$$

$$\frac{RT}{C} = \frac{R/C - r/n}{1 - r}$$

$$CRFT(x,n) = \frac{CRF(x,n) - r/n}{1 - r} \qquad (6\text{-}7)$$

$$RT = (C)CRFT(x,n) \qquad (6\text{-}8)$$

Average Annual Cost Including Taxes and Salvage

The most general model must include both the effects of taxes and of salvage value on the cost of the project. This will be accomplished by performing the same sequence of steps as in the derivation of equation (6-8). Straight-line depreciation equals (C - S)/n. Then[2]:

$$\text{Annual Taxes} = r(RT - (C-S)/n)$$

$$RT = R + \text{Annual Taxes}$$

$$RT = \frac{R}{1-r} - \frac{r(C-S)}{n(1-r)}$$

$$\frac{RT}{C-S} = \frac{R/(C-S)}{1-r} - \frac{r}{n(1-r)}$$

$$CRFST(x,n) = \frac{CRFS(x,n)}{1-r} - \frac{r}{n(1-r)} \qquad (6\text{-}9)$$

$$RT = (C-S)CRFST(x,n) \qquad (6\text{-}10)$$

Effective Average Annual Cost

The effective average annual cost will be the decision model. It is the sum of the average annual cost of the asset and the average annual operating expenses of the asset[2]:

$$AAC = (C-S)CRFST(x,n) + (E)CRF(x,n) \qquad (6\text{-}11)$$

where E is given by equation (6-2).

If taxes and salvage value are neglected, the above equation reduces to:

$$\begin{aligned} AAC &= (C)CRF(x,n) + (E)CRF(x,n) \\ &= (C + E)CRF(x,n) \\ &= (P)CRF(x,n) \end{aligned}$$

which is equation (6-4).

VII. SENSITIVITY ANALYSIS

The average annual cost model for selecting alternative projects for investment has been based on the assumption that the various input data, such as operating cost, interest rates, and salvage value, can be predicted with certainty. Under most actual conditions this would not be possible. Therefore, some method must be used to determine how errors in the input data will affect the final result.

Sensitivity analysis can be used for this purpose. It involves holding all input values constant except one; then varying that one input over its range of possible values. The resulting average annual costs can then be plotted as a function of the input variable. Some variables might be found to have little effect on the result over their possible range of values, and therefore, the risk of error arising from that source is negligible. On the other hand, some variables might prove to be highly sensitive, and it would be important to consider the risk arising from those variables.

Appendix A gives an example of the use of the average annual cost and of sensitivity analysis.

BIBLIOGRAPHY

1. Bradley, J. F. Administrative Financial Management. Holt, Rinehart and Winston, New York, 1960.

2. Dade, T. B. Economic Comparison of a Nuclear Powered and Fossil Fueled Containership. M.S. Thesis, Nuclear Engineering Department, The Pennsylvania State University, 1972.

3. Fabrycky, W. J., P. E. Torgersen. Operations Economy. Prentice-Hall, Inc., Englewood Cliffs, N.J., 1962.

4. Howell, J. E., D. Teichroew. Mathematical Analysis for Business Decisions. Richard D. Irwin, Inc., Homewood, Ill., 1963.

5. Morris, W. T. Engineering Economy. Richard D. Irwin, Inc., Homewood, Ill., 1960.

6. Sommerfield, R. M., H. M. Anderson, H. R. Brock. An Introduction to Taxation. Harcourt, Brace and World, Inc., New York, 1969.

7. Zodiaco, V. P. Economic Analysis of the Nuclear Fuel Cycle. M.S. Thesis, Nuclear Engineering Department, The Pennsylvania State University, 1971.

8. "Economics in Ship Design and Operation," Engineering Summer Conference, University of Michigan, 1966.

APPENDIX A

The following example will demonstrate the application of some of the principles discussed in this paper. The problem involves the selection of one of several pieces of machinery that performs the same function at the same rate of output. To shorten and simplify the problem, the average annual cost for only one of the machines will be calculated. The computation will be done by hand to demonstrate the procedure. In practice the average annual cost for all alternatives would be calculated using a computer. All amounts are rounded to the nearest dollar.

Reference 2 is a detailed example of the use of these principles and of sensitivity analysis.

The machine under consideration is assumed to be a special order item that requires eighteen months to produce. The manufacturer requires progress payments during the production period. The machine will have a useful life of four years and a salvage value. However, in order to collect the salvage value, the machine must be disassembled, and this cannot be completed and the money collected until one year after the shutdown of the machine. The time-money diagram for the machine is given in Figure A-1. The company is located in Pennsylvania, and has a taxable income without the machine in excess of $25,000.

The quoted price of the machine is $20,000. The manufacturer requires a down payment of $5,000, a progress payment at one year before delivery of $5,000, and final payment at delivery of $10,000.

Financing is to be accomplished through a bank loan of $5,000 at an interest rate on the unpaid balance of 8%, and the sale of

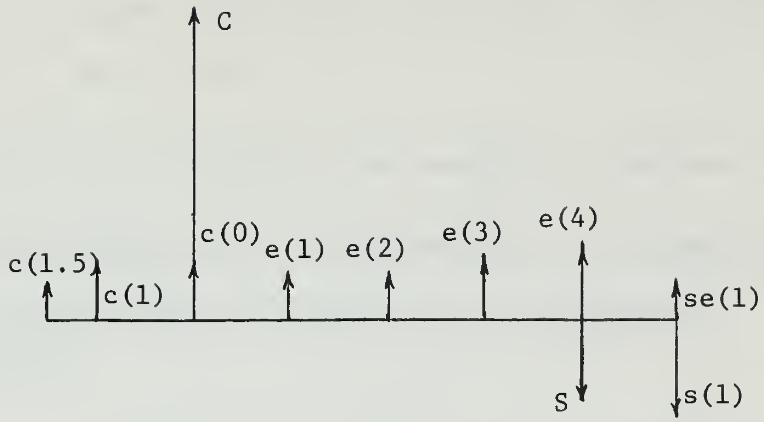

Figure A-1

Time-Money Diagram

\$15,000 worth of common stock with a cost of the common and an effective cost of capital of:

Current dividends per share	=	\$ 2.50
Retained earnings per share	=	2.00
Earnings per share	=	\$ 4.50
Market price per share	=	\$35.00
Discount and selling cost	=	5.00
Proceeds to company per share	=	\$30.00

j = Cost of common = \$4.50/\$30.00 = .15

i = Cost of loan = .08

x = Effective cost of capital = ib + j(1-b) - ibr

b = Debt to total capital ratio = \$5,000/\$20,000 = .25

r = Effective income tax rate = .61

$$x = (.08)(.25) + (.15)(.75) - (.08)(.25)(.61)$$

$$x = .12$$

The actual cost of the machine to the company at time zero is:

$$C = c(1.5)CAF(.12,1.5) + c(1)CAF(.12,1) + c(0)CAF(.12,0)$$

$$C = (5,000)(1.185) + (5,000)(1.12) + (10,000)(1)$$

$$C = \$21,525$$

The machine will incur operating expenses for the first year of operation as given in Table A-1. Also given is the expected yearly rate of escalation of these expenses due to inflation and other factors, and the resulting expense for the remaining three years.

Table A-1

Operating Expenses

Operating Expense	Escalation Rate %	Year 1	2	3	4
Wages and Fringe Benefits	6	\$10,000	\$10,600	\$11,236	\$12,022
Administrative Cost	6	200	212	225	239
Maintenance	10	500	550	605	666
Spare Parts	3	150	155	160	165
Electric Power	3	1,000	1,030	1,061	1,093
e(t)		\$11,850	\$12,547	\$13,287	\$14,185

In addition to these expenses, this machine will also require an overhaul after two years of operation. The overhaul will take one week during which time all production will stop. The machine would

have generated a revenue of \$1,000 had it not been shut down. The cost of generating that revenue would have been \$12,547/52 = \$241. The loss of income to the company will therefore be \$759. Additionally, the labor and material cost for performing the overhaul will amount to \$1,200. The alternative machines do not require this overhaul.

The expense at time zero for this machine would be:

$$E = e(1)PWF(.12,1) + e(2)PWF(.12,2) + e(3)PWF(.12,3) + e(4)PWF(.12,4) + (\text{Overhaul cost})PWF(.12,2) + (\text{Lost income cost})PWF(.12,2)$$

$$E = (11{,}850)(.89) + (12{,}547)(.80) + (13{,}287)(.71) + (14{,}185)(.64) + (1{,}200)(.80) + (759)(.80)$$

$$E = \$40{,}664$$

The cost of dismanteling the machine one year after shutdown will be \$1,500. The salvage value, s(1), at that time will be \$4,500. The net salvage value at one year after shut down, se(1), is then \$3,000. The net salvage value at shutdown is:

$$S = (3{,}000)PWF(.12,1) = (3{,}000)(.89) = \$2{,}670$$

The average annual cost of the machine can then be computed:

$$AAC = (C-S)CRFST(.12,4) + (E)CRF(.12,4)$$

$$CRFST(.12,4) = \frac{CRFS(.12,4)}{1 - .61} - \frac{.61}{4(1-.61)}$$

$$CRFS(.12,4) = CRF(.12,4) + \frac{(.12)(2{,}670)}{(21{,}525-2{,}670)}$$

$$\text{CRFS}(.12,4) = .3292 + .0169$$

$$\text{CRFS}(.12,4) = .3461$$

$$\text{CRFST}(.12,4) = \frac{.3461}{.39} - \frac{.61}{1.56}$$

$$\text{CRFST}(.12,4) = .4964$$

$$\text{AAC} = (21{,}525-2{,}670)(.4964) + (40{,}664)(.3292)$$

$$\underline{\text{AAC} = \$22{,}746}$$

This cost would be compared to that of the alternative machines and the lowest cost machine recommended. It would be advisable to conduct a sensitivity analysis on various input data, such as the lost income during the overhaul, the escalation rate of the operating costs, the cost of capital, etc. A sensitivity analysis of the salvage value will be performed as an example. The results are plotted in Figure A-2.

The possible range of the net salvage value at time of shutdown, S, is from \$1,000 to \$4,000. The average annual cost for various values of salvage value would be: S = \$1,000, AAC = \$22,972; S = \$3,000, AAC = \$22,701; S = \$4,000, AAC = \$22,587.

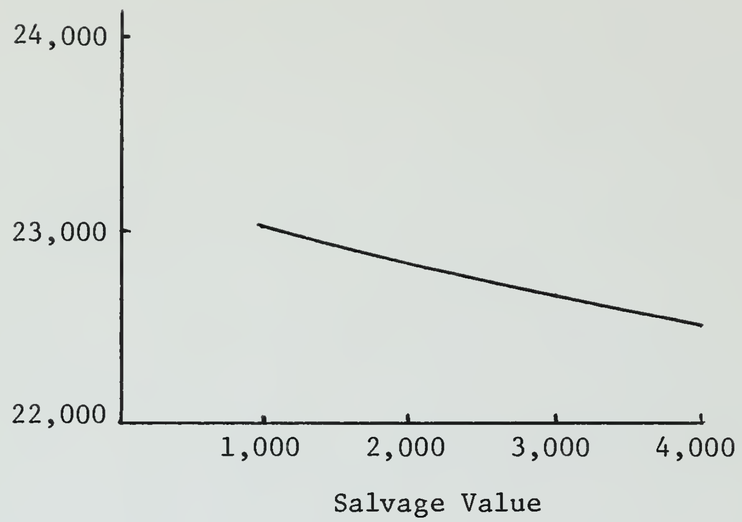

Figure A-2

Average Annual Cost Versus Salvage Value

The graph shows that the average annual cost is only moderately effected by the salvage value. Therefore, the risk of an error in estimating the salvage value is not too great.

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